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Model independent calculation of $\mathcal{B}(\bar{B}^0 \to D^{(*)+}\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^{(*)+}e^-\bar{\nu})$

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Abstract. Using the formulas for the $\mathrm{d}\Gamma/\mathrm{d}q^2$ distribution with non-zero lepton mass and experimentally determined form factors, we calculate the $\mathrm{d}\Gamma(D^{(*)+}l^-\bar{\nu})/\mathrm{d}q^2$ spectra and branching fractions for $l = e$, μ and τ. We obtain the results $\mathcal{B}(\bar{B}^0 \to D^+\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^+e^-\bar{\nu}) = 0.278^{+0.049}_{-0.035}$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^{*+}e^-\bar{\nu}) = 0.256^{+0.014}_{-0.013}$. Since we used the experimentally measured form factors, these results are independent of theoretical models of the form factors.

The heavy quark effective theory (HQET) allows the form factors of the heavy to heavy meson transitions to be expressed in terms of the Isgur–Wise function [1]. The HQET can be applied to the B to D transitions, and the precise measurements of the q^2 spectra of the exclusive semileptonic decays $\bar{B}^0 \to D^{(*)+}l^-\bar{\nu}$ with an electron or muon have been used for the determination of the Isgur–Wise function [2,3]. However, the exclusive semileptonic decays with tau lepton have not been measured accurately yet, and these measurements will be interesting in the B-factories. Even though these branching fractions are not measured directly, ALEPH estimated $\mathcal{B}(\bar{B}^0 \to D^+\tau^-\bar{\nu}) = (0.69 \pm 0.14)\%$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}) = (2.06 \pm 0.41)\%$ [4] from their experimental result $\mathcal{B}(b \to \tau^-\bar{\nu}X) = (2.75 \pm 0.30 \pm 0.37)\%$ [5] of the inclusive decays by assuming that one-fourth of $b \to \tau^-\bar{\nu}X$ involve a D^+ meson and the other three-fourths involve a D^{*+} meson. The above ALEPH estimations correspond to $\mathcal{B}(\bar{B}^0 \to D^+\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^+e^-\bar{\nu}) = 0.29 \pm 0.08$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^{*+}e^-\bar{\nu}) = 0.37 \pm 0.08$. ALEPH used these estimations for the background rejection in the measurement of $|V_{cb}|$ from $\bar{B}^0 \to D^{(*)+}l^-\bar{\nu}$ decays [4]. OPAL also used those estimations in the measurement of $|V_{cb}|$ from $\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}$ decays [6]. On the other hand, the result of L3 for the inclusive decays is given by $\mathcal{B}(b \to \tau^-\bar{\nu}X) = (1.7 \pm 0.5 \pm 1.1)\%$ [7]. In this paper we calculate the branching fractions of the exclusive semileptonic decays $\bar{B}^0 \to D^{(*)+}\tau^-\bar{\nu}$ within the HQET by using the experimentally measured form factors without using models for the form factors. Therefore, our calculation is model independent and does not contain any theoretical ambiguities coming from form factor models. We obtain $\mathcal{B}(\bar{B}^0 \to D^+\tau^-\bar{\nu}) = (0.52 \pm 0.07)\%$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu}) = (1.22 \pm 0.06)\%$, which corresponds to $\mathcal{B}(\bar{B}^0 \to D^+\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^+e^-\bar{\nu}) = 0.278^{+0.049}_{-0.035}$ and $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^{*+}e^-\bar{\nu}) = 0.256^{+0.014}_{-0.013}$. Our result for the second ratio is smaller than the corresponding ALEPH estimation, and the ratio $\mathcal{B}(\bar{B}^0 \to D^+\tau^-\bar{\nu}) : \mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu})$ is about $1 : 2.35$ instead of $1 : 3$. The sum of our results of the two exclusive modes is also smaller than the ALEPH value of the inclusive decays, and it is close to the mean value of the L3 result. Körner and Schuler also studied the decays $\bar{B}^0 \to D^{(*)+}\tau^-\bar{\nu}$ by using the form factors from the free quark decay model and the spectator quark model [8]. Tanaka studied charged Higgs effects in the semi-tauonic B decays and found that the measurements of these decays will give non-trivial constraints on the charged Higgs sector [9]. In the present paper, we use the recent experimental results [2,3] for the form factors.

From Lorentz invariance one finds the decomposition of the hadronic matrix element in terms of the hadronic form factors:

$$\begin{aligned}
&\langle D^+(p)|J_\mu|\bar{B}^0(P)\rangle \\
&= (P+p)_\mu f_+(q^2) + (P-p)_\mu f_-(q^2) \\
&= \left((P+p)_\mu - \frac{M^2-m^2}{q^2}q_\mu\right)F_1(q^2) \\
&+\frac{M^2-m^2}{q^2}q_\mu F_0(q^2),
\end{aligned} \tag{1}$$

where $J_\mu = \bar{c}\gamma_\mu(1-\gamma_5)b$. We use the following notation: $M = m_B$ represents the initial meson mass, $m = m_{D^{(*)}}$ the final meson mass, m_l the lepton mass, $P = p_B$, $p = p_{D^{(*)}}$, and $q_\mu = (P-p)_\mu$. The form factors $F_1(q^2)$ and $F_0(q^2)$ correspond to 1^- and 0^+ exchanges, respectively. At $q^2 = 0$ we have the constraint $F_1(0) = F_0(0)$, since the hadronic matrix element in (1) is non-singular at this kinematic point.

The differential decay rate is given by

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{M}|^2\mathrm{d}q^2\mathrm{d}t, \quad \mathcal{M} = \frac{G_\mathrm{F}}{\sqrt{2}}V_{cb}l^\mu h_\mu, \quad (2)$$

where $l^\mu = \bar{u}_l\gamma^\mu(1-\gamma_5)v_\nu$ and $h_\mu = \langle D^+(p)|J_\mu|\bar{B}^0(P)\rangle$. We use the notation $k = p_{l^-}$, $\bar{k} = p_{\bar{\nu}}$, $q^2 = (P-p)^2 = (k+\bar{k})^2$, $t = (P-\bar{k})^2 = (p+k)^2$, and $u = (P-k)^2 = (p+\bar{k})^2$. From (2) we have

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\mathrm{d}t} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}\frac{G_\mathrm{F}^2|V_{cb}|^2}{2}l_{\mu\nu}h^{\mu\nu}, \quad (3)$$

where

$$l_{\mu\nu} = 4(k_\mu\bar{k}_\nu + \bar{k}_\mu k_\nu - g_{\mu\nu}k\cdot\bar{k} - \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}k^\rho\bar{k}^\sigma), \quad (4)$$
$$h_{\mu\nu} = \langle D^+(p)|J_\mu|\bar{B}^0(P)\rangle^*\langle D^+(p)|J_\mu|\bar{B}^0(P)\rangle. \quad (5)$$

From (1), (4) and (5), after some calculations we get

$$\begin{aligned}\frac{1}{8}l_{\mu\nu}h^{\mu\nu} &= |f_+(q^2)|^2\Big(-t^2 + t(M^2+m^2-q^2) \\ &\quad - M^2m^2 + m_l^2\Big(t + \frac{1}{4}(q^2-m_l^2)\Big)\Big) \\ &\quad + \mathrm{Re}(f_+(q^2)f_-(q^2))\frac{1}{2}m_l^2(2t+q^2-2m^2-m_l^2) \\ &\quad + |f_-(q^2)|^2\frac{1}{2}m_l^2\frac{1}{2}(q^2-m_l^2). \quad (6)\end{aligned}$$

The allowed range of the variable t is given by

$$\begin{aligned}ME - \frac{m_l^2}{q^2}(ME-m^2) - MK\left(1-\frac{m_l^2}{q^2}\right) &\le t \\ \le ME - \frac{m_l^2}{q^2}(ME-m^2) + MK\left(1-\frac{m_l^2}{q^2}\right), \quad (7)\end{aligned}$$

where the energy E and the magnitude of three momentum K of the $D^{(*)+}$ meson in the $\bar{B}^0$ meson rest frame are given by

$$E(q^2) = \frac{1}{2M}(M^2+m^2-q^2),$$
$$K(q^2) = \frac{1}{2M}((M^2+m^2-q^2)^2 - 4M^2m^2)^{1/2}. \quad (8)$$

After the t integration of (3) over the allowed range given in (7), the q^2 distribution of the decay rate is given by

$$\begin{aligned}\frac{\mathrm{d}\Gamma(\bar{B}^0\to D^+l^-\bar{\nu})}{\mathrm{d}q^2} &= \frac{G_\mathrm{F}^2}{24\pi^3}|V_{cb}|^2K(q^2)\left(1-\frac{m_l^2}{q^2}\right)^2 \quad (9) \\ &\quad\times\Bigg[(K(q^2))^2\left(1+\frac{1}{2}\frac{m_l^2}{q^2}\right)|F_1(q^2)|^2 \\ &\quad + M^2\left(1-\frac{m^2}{M^2}\right)^2\frac{3}{8}\frac{m_l^2}{q^2}|F_0(q^2)|^2\Bigg],\end{aligned}$$

where the allowed range of q^2 is given by

$$m_l^2 \le q^2 \le (M-m)^2. \quad (10)$$

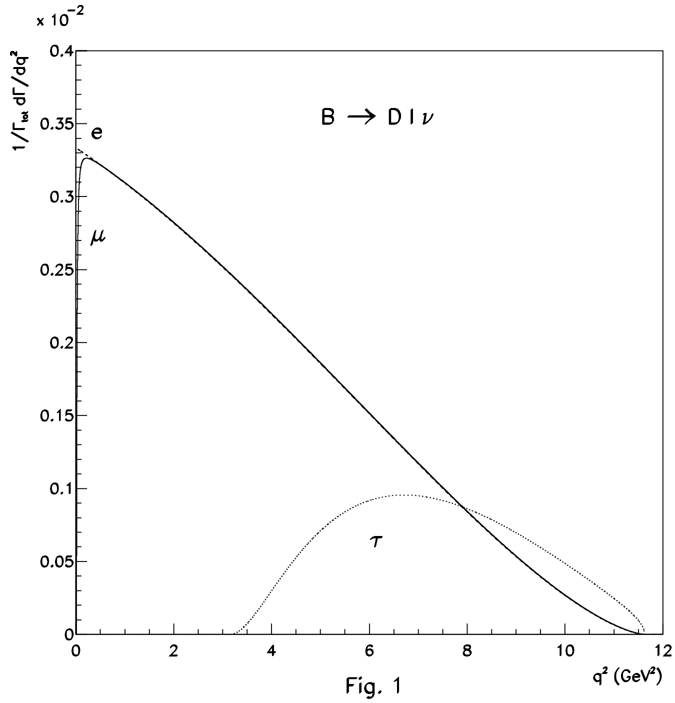

Fig. 1. $(1/\Gamma_\mathrm{tot})(\mathrm{d}\Gamma/\mathrm{d}q^2)$ of $\bar{B}^0 \to D^+l^-\bar{\nu}$

The formula (9) was also given by Khodjamirian et al. [10]. For $m_l = 0$, (9) is reduced to the well-known formula $\mathrm{d}\Gamma/\mathrm{d}q^2 = (G_\mathrm{F}^2/(24\pi^3))|V_{cb}|^2(K(q^2))^3|F_1(q^2)|^2$ and $0 \le q^2 \le (M-m)^2$.

From the HQET, we have [1]

$$\begin{aligned}F_1(q^2) = V(q^2) &= A_0(q^2) \\ &= A_2(q^2) = \frac{M+m}{2\sqrt{Mm}}\mathcal{F}(y), \quad (11) \\ F_0(q^2) = A_1(q^2) &= \frac{2\sqrt{Mm}}{M+m}\frac{y+1}{2}\mathcal{F}(y),\end{aligned}$$

where $y = (M^2+m^2-q^2)/(2Mm)$. We use the experimentally measured results [2,3]

$$\mathcal{F}(y) = \mathcal{F}(1)[1-\rho^2(y-1)] \quad \text{with} \quad (12)$$
$$\begin{cases}\rho_D^2 = 0.59\pm0.25, \ |V_{cb}|\mathcal{F}_D(1)\times\ 10^2 = 3.37 \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{for } \mathcal{F}_D(y) \\ \rho_{D^*}^2 = 0.84\pm0.15, \ |V_{cb}|\mathcal{F}_{D^*}(1)\times\ 10^2 = 3.51 \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{for } \mathcal{F}_{D^*}(y),\end{cases}$$

where we used the mean values of $|V_{cb}|\mathcal{F}_{D^{(*)}}(1)$ since the ratios of branching fractions are independent of these normalizations. The difference of the numerical values of the parameters for D and D^* in (12) reflects the fact that the heavy quark mass limit is not complete in the B to $D^{(*)}$ transitions.

By using the form factors in (11) with $\mathcal{F}_D(y)$ in (12), we obtain from (9) the spectra presented in Fig. 1, where we find that the spectrum for muon drops down near $q^2 = 0$. (The spectrum for the electron also drops down near the very end of $q^2 = 0$.) In Fig. 2 we can see this characteristic

Table 1. The obtained branching fractions and their ratio for $\bar{B}^0 \to D^{(*)+}l^-\bar{\nu}$

	$\mathcal{B}(D^{(*)+}e^-) \times 10^2$	$\mathcal{B}(D^{(*)+}\mu^-) \times 10^2$	$\mathcal{B}(D^{(*)+}\tau^-) \times 10^2$	$\mathcal{B}(e^-) : \mathcal{B}(\mu^-) : \mathcal{B}(\tau^-)$
D^+	$1.85^{-0.47}_{+0.56}$	$1.84^{-0.47}_{+0.56}$	$0.52^{-0.07}_{+0.07}$	$1 : 0.996^{+0.001}_{-0.001} : 0.278^{+0.049}_{-0.035}$
D^{*+}	$4.76^{-0.47}_{+0.50}$	$4.74^{-0.46}_{+0.50}$	$1.22^{-0.06}_{+0.06}$	$1 : 0.996^{+0.002}_{-0.000} : 0.256^{+0.014}_{-0.013}$

in more detail. At $q^2 = m^2_{\pi^-} = 0.019\,\mathrm{GeV}^2$, $\mathrm{d}\Gamma/\mathrm{d}q^2$ for a muon has a substantially smaller value than that for an electron, and the formula $[\Gamma(\bar{B}^0 \to D^+\pi^-)]/[\mathrm{d}\Gamma(\bar{B}^0 \to D^+l^-\bar{\nu})/\mathrm{d}q^2|_{q^2=m^2_{\pi^-}}] = 6\pi f_\pi^2|a_1|^2|V_{ud}|^2 X_\pi$ which is used for the test of the factorization holds for the electron spectrum under the factorization assumption, but not for the muon spectrum if we look at the $\mathrm{d}\Gamma/\mathrm{d}q^2$ spectrum in full detail. We present the obtained branching fractions and their ratio in Table 1, which gives $\mathcal{B}(\bar{B}^0 \to D^+\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^+e^-\bar{\nu}) = 0.278^{+0.049}_{-0.035}$. Since the HQET provides good information about the heavy to heavy form factors; our results for the exclusive B to D semileptonic decays are reliable without theoretical model dependence.

From Lorentz invariance one finds the decomposition of the hadronic matrix element in terms of hadronic form factors:

$$\langle D^{*+}(p)|J_\mu|\bar{B}^0(P)\rangle = \varepsilon^{*\nu}(p)\Bigg((M+m)g_{\mu\nu}A_1(q^2) - 2\frac{P_\mu P_\nu}{M+m}A_2(q^2) + \frac{q_\mu P_\nu}{M+m}A_3(q^2) + \mathrm{i}\varepsilon_{\mu\nu\rho\sigma}\frac{P^\rho p^\sigma}{M+m}V(q^2)\Bigg), \tag{13}$$

where $\varepsilon_{0123} = 1$ and

$$2mA_0(q^2) = (M+m)A_1(q^2) - \frac{M^2-m^2+q^2}{M+m}A_2(q^2) + \frac{q^2}{M+m}A_3(q^2). \tag{14}$$

The form factors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$ and $A_0(q^2)$ correspond to 1^-, 1^+, 1^+ and 0^- exchanges, respectively. At $q^2 = 0$ we have the constraint $2mA_0(0) = (M+m)A_1(0) - (M-m)A_2(0)$, since the hadronic matrix element in (13) is non-singular at this kinematic point.

After a rather lengthy calculation similar to the procedure from (2) to (9) for $\bar{B}^0 \to D^+l^-\bar{\nu}$ decays, we find that the q^2 distribution of the decay rate for $\bar{B}^0 \to D^{*+}l^-\bar{\nu}$ is given by

$$\begin{aligned}
\frac{\mathrm{d}\Gamma(\bar{B}^0 \to D^{*+}l^-\bar{\nu})}{\mathrm{d}q^2} &= \frac{G_\mathrm{F}^2}{32\pi^3} \\
&\times |V_{cb}|^2\frac{1}{M^2}K(q^2)\left(1-\frac{m_l^2}{q^2}\right)^2 \\
&\times\Bigg\{|A_1(q^2)|^2\frac{(M+m)^2}{m^2}\left[\frac{1}{3}(MK)^2\left(1-\frac{m_l^2}{q^2}\right) + q^2m^2 + (MK)^2\frac{m_l^2}{q^2} + \frac{1}{2}m^2m_l^2\right] \\
&+\mathrm{Re}(A_1(q^2)A_2^*(q^2))\Bigg[-\frac{M^2-m^2-q^2}{m^2} \\
&\times\left[\frac{2}{3}(MK)^2\left(1-\frac{m_l^2}{q^2}\right) + 2(MK)^2\frac{m_l^2}{q^2} + \frac{1}{2}(M^2+m^2-q^2)m_l^2\right] + (M^2-m^2+q^2)m_l^2\Bigg] \\
&+|A_2(q^2)|^2\frac{1}{(M+m)^2m^2}(MK)^2\left[\frac{4}{3}(MK)^2\left(1-\frac{m_l^2}{q^2}\right) + 4(MK)^2\frac{m_l^2}{q^2} + 2M^2m_l^2\right] + |V(q^2)|^2\frac{q^2}{(M+m)^2} \\
&\times\left[\frac{8}{3}(MK)^2\left(1-\frac{m_l^2}{q^2}\right) + 4(MK)^2\frac{m_l^2}{q^2}\right] \\
&+|A_3(q^2)|^2\frac{q^2}{(M+m)^2m^2}\frac{1}{2}(MK)^2m_l^2 \\
&-\mathrm{Re}(A_3(q^2)A_2^*(q^2))\frac{1}{(M+m)^2m^2} \\
&\times(M^2-m^2+q^2)(MK)^2m_l^2 \\
&+\mathrm{Re}(A_3(q^2)A_1^*(q^2))\frac{1}{m^2}(MK)^2m_l^2\Bigg\}.
\end{aligned} \tag{15}$$

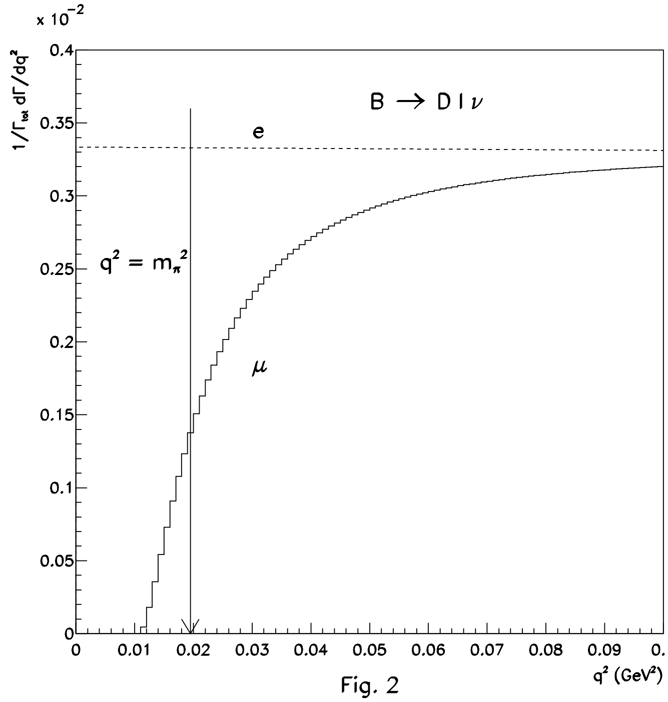

Fig. 2. $(1/\Gamma_\mathrm{tot})(\mathrm{d}\Gamma/\mathrm{d}q^2)$ of $\bar{B}^0 \to D^+l^-\bar{\nu}$ for $0 \le q^2 \le 0.1\,\mathrm{GeV}^2$

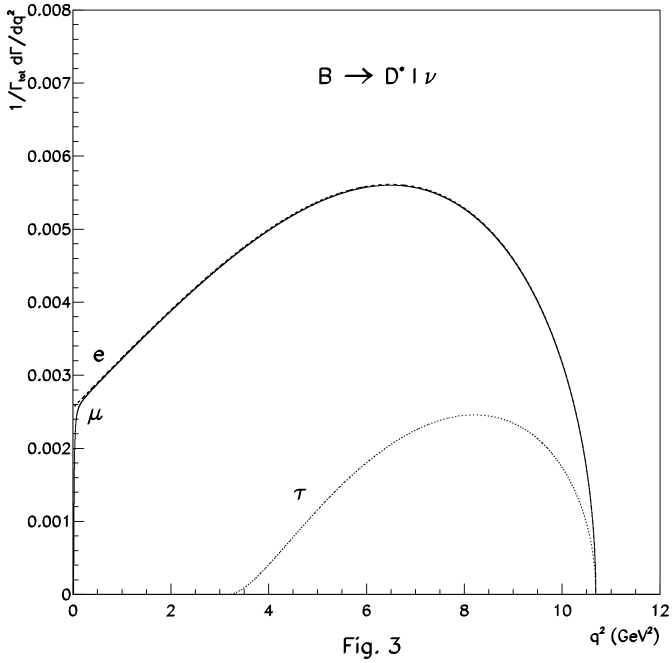

Fig. 3. $(1/\Gamma_{\rm tot})(\mathrm{d}\Gamma/\mathrm{d}q^2)$ of $\bar{B}^0 \to D^{*+}l^-\bar{\nu}$

When $m_l = 0$ is taken in the formula (15), it agrees with the formula for $m_l = 0$ given in [11], and when we use the relations (11) of the HQET with $m_l = 0$, (15) reduces to the well-known formula $\mathrm{d}\Gamma/\mathrm{d}q^2 = (G_{\rm F}^2/(48\pi^3))|V_{cb}|^2 m^3 \times (M-m)^2(y^2-1)^{1/2}(y+1)^2\{1+(4y/(y+1))((1-2yr+r^2)/(1-r)^2)\}(\mathcal{F}_{D^*}(y))^2$, where $r = m/M$.

By using the form factors given in (11) with $\mathcal{F}_{D^*}(y)$ in (12), we obtain from (15) the spectra presented in Fig. 3. We present the obtained branching fractions and their ratio in Table 1, where we find that $\mathcal{B}(\bar{B}^0 \to D^{*+}\tau^-\bar{\nu})/\mathcal{B}(\bar{B}^0 \to D^{*+}e^-\bar{\nu}) = 0.256^{+0.014}_{-0.013}$ which is smaller than the ALEPH estimation 0.37 ± 0.08 [4].

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